QP CODE: 26019089

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Reg No : .....

Name : .....

M.Sc DEGREE (CSS) EXAMINATION, MARCH 2026

Fourth Semester

M Sc MATHEMATICS

ELECTIVE - ME800402 - ALGORITHMIC GRAPH THEORY

2019 Admission Onwards

2FDE000B

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Prove that every graph contains an even number of odd vertices.
2. (a) Explain a bridge and a cut vertex in a graph with example.
   (b) Find a counter example to the statement,' if G is a connected graph with a cut-vertex, then G contains a bridge'.
3. Define floor and ceiling of a number with examples.
4. Prove that every complete binary tree with i internal vertices has $i + 1$ leaves.
5. Show that the distance function on a graph G is a metric.
6. Show that for a graph G , $rad\ G \leq diam\ G \leq 2\ rad\ G$
7. Define edge connectivity and vertex connectivity of a graph. Give an example of a graph with $\kappa(G) \neq \lambda(G) \neq \delta(G)$
8. Define the terms $M(u, v)$ and $M'(u, v)$
9. *Describe the optimal assignment problem*
10. *Define a BIBD*

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Explain with examples (a) strongly connected digraph (b) unilaterally connected digraph (c) weakly connected digraph (d) symmetric digraph (e) asymmetric digraph.

12. Draw the graph G with
$V(G) = \{v_1, v_2, v_3, v_4, v_5\}$ $E(D) = \{v_1v_2, v_1v_3, v_2v_3, v_1v_4, v_2v_5, v_4v_5, v_4v_3\}$.Determine its adjacency matrix, adjacency list, adjacency list table

13. If u and v are distinct vertices of a tree T, then prove that T contains exactly one $u - v$ path.

14. Define the minimum spanning tree problem. Write an algorithm to determine a minimum spanning tree in a non - trivial connected weighted graph.

15. Give an example of a flow in a network where flow along each arc is a positive integer and find the value of the flow.

16. Write the Max-Flow Min-Cut Algorithm.

17. *Define an alternating tree. Write the maximum matching algorithm for bipartite graphs*

18. *Prove that every regular bipartite multigraph of degree $r \geq 1$ is 1-factorable*

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. a) Havel – Hakimi Theorem
b) Determine whether the following sequence is graphical and , if so, find a graph having s as its degree sequence.
$s : 5, 5, 4, 4, 3, 3, 3, 1, 0, 0$

20. Explain Breadth First Search Algorithm using an example. Find its complexity.

21. State and prove Ford and Fulkerson Theorem.

22. *a) Define a Hamiltonian graph with example*
b) Prove that for every positive integer n , the graph K_{2n+1} can be factored into n Hamiltonian cycles
c) Give an ascending subgraph decomposition of the Petersen graph

(2×5=10 weightage)